Investigation of granular flow using silo centrifuge models

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Tuesday 22 October 2013

Introduction and motivation

A silo centrifuge model has been developed to investigate silo flow behaviour at different gravities.

- Many features of silo design are only partially understood, even though discharge behaviours have been investigated for over a century.
- Empirical and phenomenological models are often used to facilitate silo design.
- A lack of analytical models is associated with inefficient processes.

Aims and objectives

- Understanding the effect of stress level on flow behaviour
- Establishing the scaling laws governing this behaviour

Investigate:

- Influence of gravity on flow rate
- Compare Beverloo correlation to observations at different gravities
- Influence of material properties on flow rate response to increased gravity

Centrifuge modelling background

- Widely used in geotechnical engineering
- Early silo centrifuge models in 1970's
  - Computational and instrumentation limitations
- Scaled silo centrifuge model produces same stresses and strains in same relative locations as prototype scale (according to continuum theory)
- Quicker and cheaper than prototype scale
- Higher stresses than reduced scale models in 1g environment

Theoretical background - stress equivalence

$$q_{prototype} = \frac{1}{\mu K}\frac{A}{U}\rho_b g\left(1 - e^{-z\Big/\frac{1}{\mu K}\frac{A}{U}}\right) \tag{1}$$

$$q_{model} = \frac{1}{\mu K}\frac{A}{N^2}\frac{N}{U}\rho_b N g\left(1 - e^{-z\Big/\frac{1}{\mu K}\frac{A}{N^2}\frac{N^2}{U}}\right) \tag{2}$$

$$\therefore \quad q_{prototype} = q_{model}$$

Geotechnical centrifuge

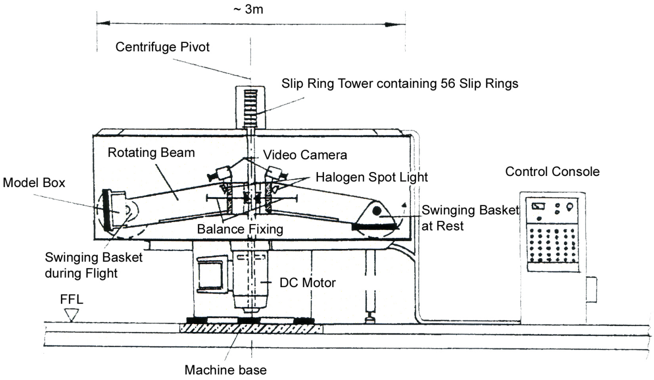


Figure 1: Schematic sketch of Trio-Tech 1231 Geotechnical Centrifuge

Table 1: Centrifuge specifications (TRIO-TECH, 1988)

Property	Value
Diameter of centrifuge [m]	3.0
Radius of swinging basket axis [m]	1.085
Motor	15HP DC
Slip rings	56
Radial acceleration [g]	0 to 200
Rotations per minute [1/min]	0 to 400
Maximum load capacity [G-kg]	10,000
Maximum model mass [kg]	90
Maximum model dimensions WxDxH [mm]	540 x 560 x 560
Total weight [kg]	2041

Design criterion

Modelling requirements

- More than 100 particle diameters wide
- Internal wall surfaces should be smooth
- Filling should be standardised
- Quasi-planar
- Height should be maximised

Research requirements

- Model silo must be observable
- Model silo should facilitate as many kinds of experiments as possible
- Various granular materials should be able to be used
- Adequate space for data loggers, camera, lights, etc.

Silo centrifuge model

Four materials tested

Table 2: Material properties

Property	Fine sand	Coarse sand	Glass beads	Polyamide
Particle Diameter $D_{50}/\ d_1, d_2$ [mm]	0.4	0.8	3.15 ± 0.1, 1.45 ± 0.1	0.75 ± 0.1, 1.5 ± 0.1
Particle density ρ_s $[g/cm^3]$	2.65	2.644	2.750	1.1
Bulk density ρ_b $[g/cm^3]$	1.4 - 1.6	1.44 - 1.65	1.52	0.65
Void ratio e [-]	1.5	1.4	0.809	0.692
Friction angle θ_i $[^\circ]$	34	34	22	25
Cohesion c $[kN/m^2]$	0	0	0	0

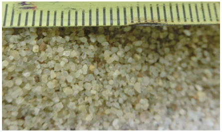


Figure 2:Fine sand

Figure 3: Coarse sand

Figure 4: Glass beads mixture

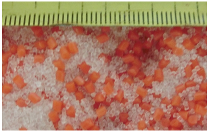


Figure 5: Polyamide mixture

Instrumentation

- Load cells record the mass of discharging material entering a collection bucket beneath the silo
- High-speed video records flow behind the front transparent acrylic wall (512 × 384 pixels, 232fps)
- Particle Image Velocimetry analysis is used to quantify the flow fields during discharge
- Pressure pads map the pressure distribution on lateral walls before and during discharge

Glass beads, 5g

Glass beads, 15g

11.5x slower (20 fps, original = 232 fps)

Settlement of material M2: Coarse sand

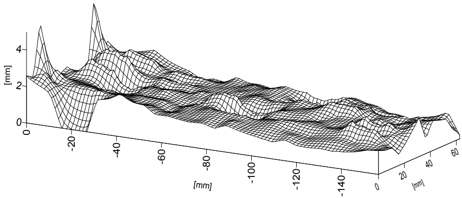


Figure 6: 10g

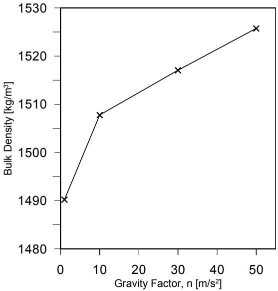


Figure 8: Density increase as a result of increased gravity

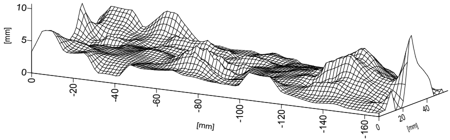


Figure 7: 50g

Pressure pad results, model silo with 60° hopper, coarse sand (M2)

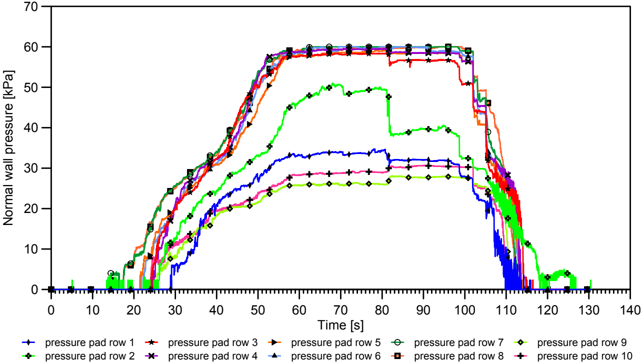


Figure 9: Silo wall pressures, coarse sand in silo with 60° hopper at 50g

Pressure pad results in model silo with 60 degree hopper

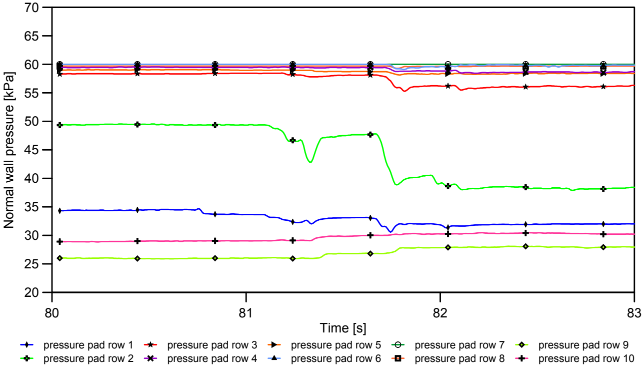


Figure 10: Silo wall pressures, coarse sand in silo with 60° hopper at 50g

result

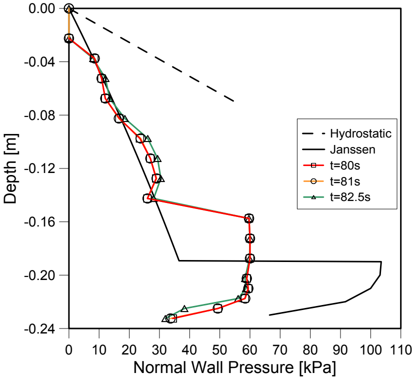


Figure 11: Normal wall pressures at 3 times. LHS, coarse sand in silo with 60 degree hopper at 50g

Beverloo correlation

Flat-bottomed silos:

$$W_B = C\rho_b\sqrt{g^*}(l - kd)(D - kd)^{1.5}$$

W_B = mass flow rate (kg/s)
l = long dimension of outlet
D = small dimension of outlet
ρ_b = Bulk density

$k = 1$
g^* = applied gravity
$C = 1.03$
d = Average grain diameter

Silos with hopper:

when $\beta < 90 - \phi_d$: $W \propto (\tan\beta \tan\phi_d)^{-0.35} \quad \rightarrow \quad W = W_B F(\beta, \phi_d)$

where ϕ_d is the angle between the stagnant zone boundary and the horizontal, β is the hopper half angle. detail

Glass beads

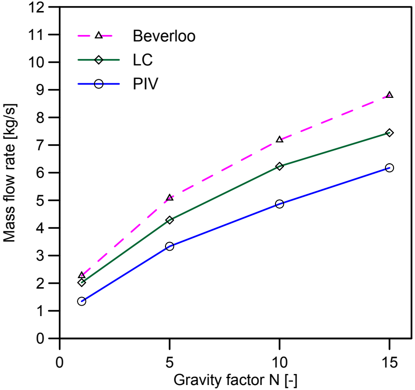


Figure 12: Flat bottomed silo

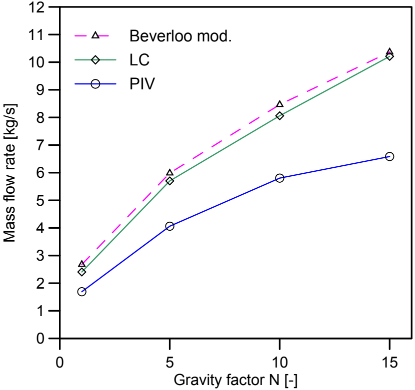


Figure 13: Silo with 30° hopper

LC - Load cells **PIV - Particle image velocimetry**

Glass beads

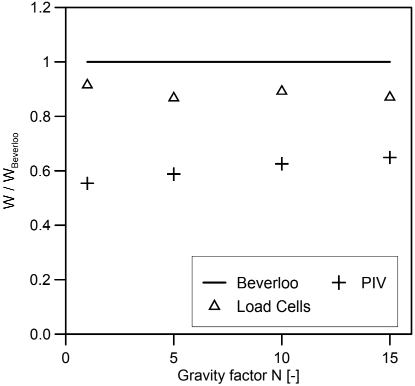


Figure 14: Silo with flat bottom

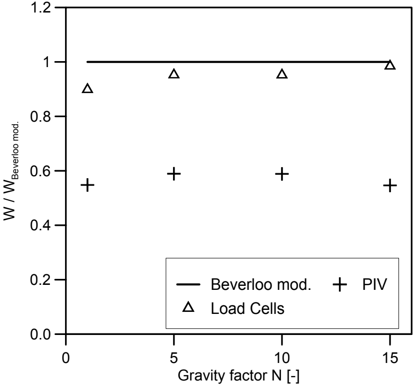


Figure 15: Silo with 30° hopper

LC - Load cells **PIV - Particle image velocimetry**

Discharge time

$$\frac{t}{t_0} = \sqrt{\frac{g}{g_0}} \quad \rightarrow \quad t = t_0\sqrt{\frac{g}{g_0}} \quad \rightarrow \quad t_m = t_p N^{-1/2} \qquad (3)$$

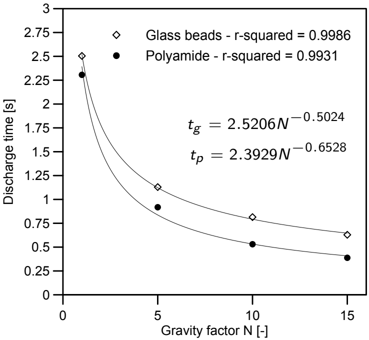


Figure 16: Flat bottomed silo

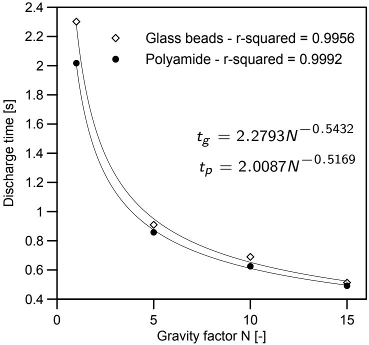
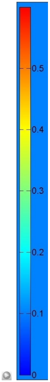


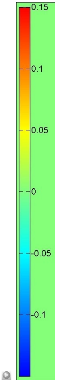
Figure 17: Silo with 30° hopper

Glass beads, 5g, $W_0 = 30mm$

Vertical component $[m/s]$



Horizontal component $[m/s]$



19x slower (12fps, original = 232 fps)

PIV methodology

- The average flow field was calculated between 10% and 40% of discharge.
- The velocity distribution along a horizontal line 112*mm* above the silo outlet was investigated.

Figure 18: Line 112mm above outlet showing position of velocity profile

Glass beads

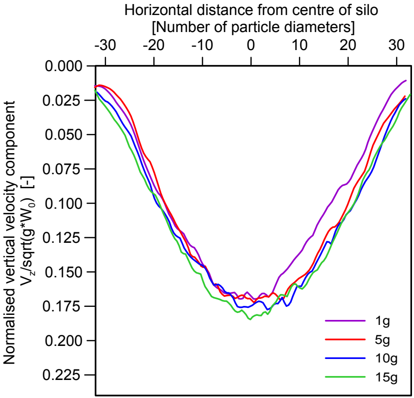


Figure 19: Silo with flat bottom

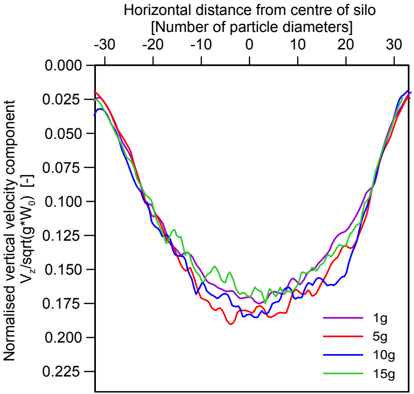


Figure 20: Silo with 30° hopper

Glass beads

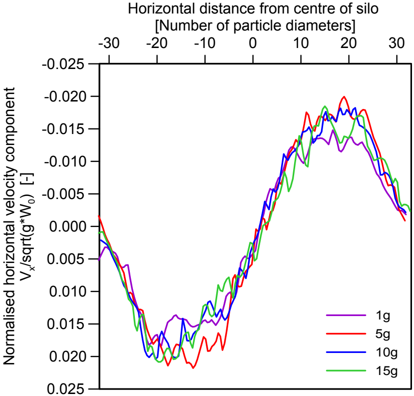


Figure 21: Silo with flat bottom

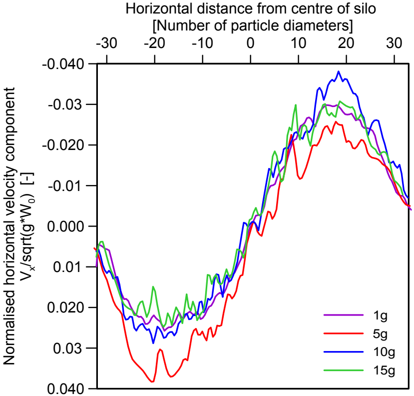


Figure 22: Silo with 30° hopper

Glass beads, Flat bottomed silo

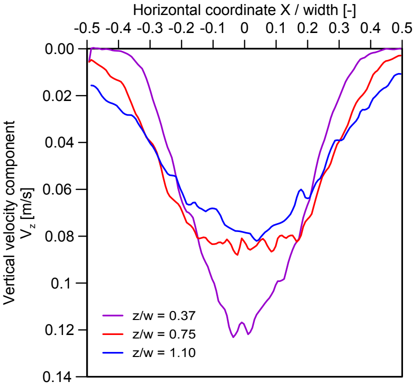


Figure 23: 1g

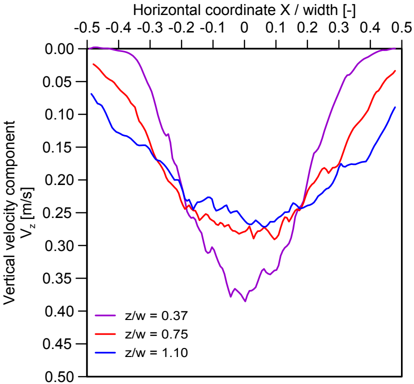


Figure 24: 10g

Glass beads, 10g

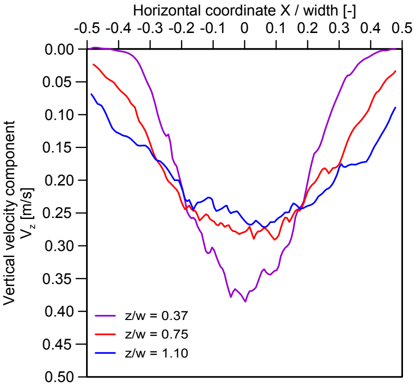


Figure 25: Silo with flat bottom

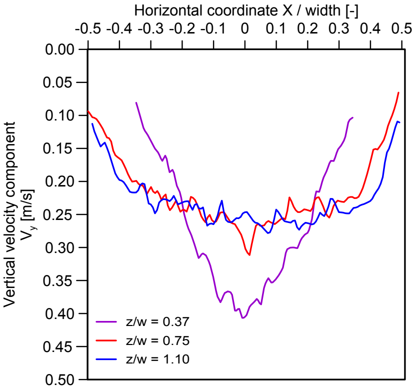


Figure 26: Silo with 30 degree hopper

Particle size distribution

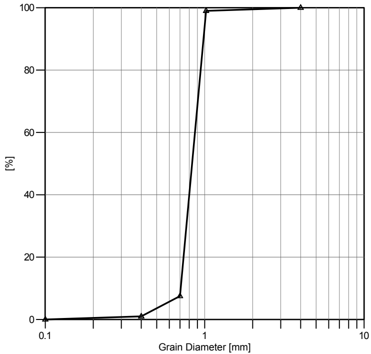


Figure 27: Particle size distribution of material M2, DIN 1164/58 Norm Sand II Klein (1998)

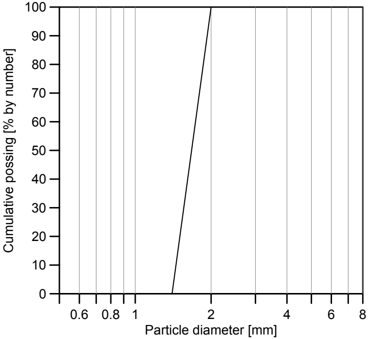


Figure 28: Particle size distribution in numerical model

DEM parameters DEM settlement

Triaxial calibration

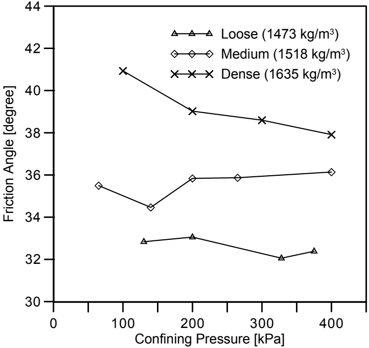


Figure 29: Variation of friction angle with confining pressure for physical samples of different initial density

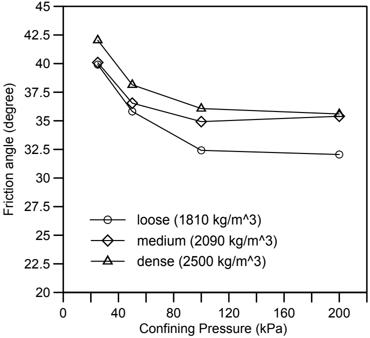


Figure 30: Variation of friction angle with confining pressure for DEM samples of different initial density

Discharge rates - silo with flat bottom

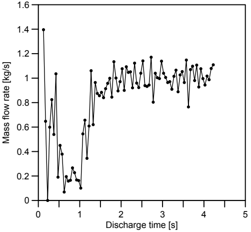


Figure 31: 50g

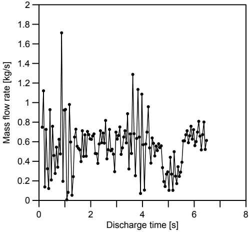


Figure 33: 30g

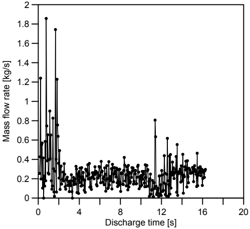


Figure 35: 10g

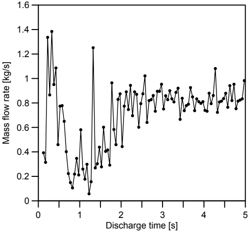


Figure 32: 40g

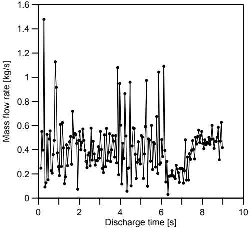


Figure 34: 20g

Discharge rate comparison

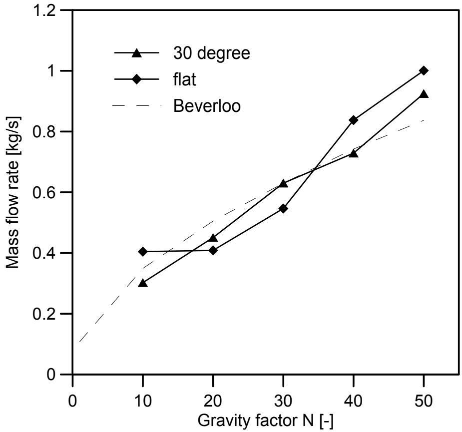


Figure 36: Observed discharge rates compared with Beverloo prediction

Conclusions

- Quasi-two-dimensional silo centrifuge model developed
- Four materials tested - Fine sand, Coarse sand, Glass beads and Polyamide
- Two silo geometries tested - 30° hopper and flat bottom

- Discharge rate is proportional to square root of gravity
- Internal flow velocity is proportional to square root of gravity
- Stagnant zone boundaries are independent of gravity
- Friction angle is independent of gravity

Thank you

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References I

Rose, H. F. and T. Tanaka (1956). In: *The Engineer (London)*, page 208.

Beverloo, W. A., H. A. Leniger, and J. van de Velde (1961). "The Flow of Granular Solids Through Orifices". In: *Chemical Engineering Sciences*, pages 260 –269.

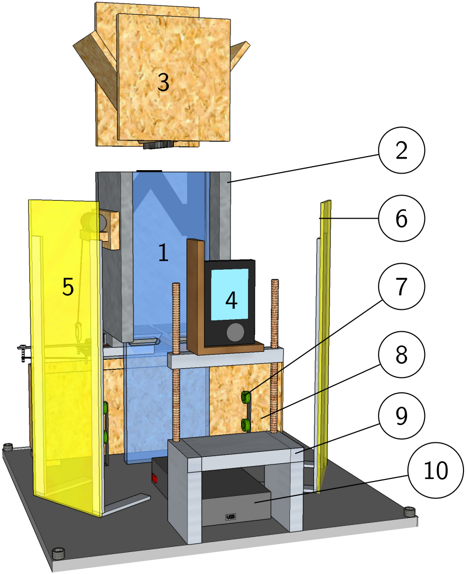
Supplemental content

Back to beginning

Theoretical background - stress equivalence

- Treats granular media as continuous.
- Predicts that a 1/N scale centrifuge model will produce the same stresses and strains in the same relative locations as in a prototype.

$$\textbf{scale } 1/\mathrm{N} \Longrightarrow \begin{cases} \textbf{Acceleration} \rightarrow \textbf{Acceleration} \times N \\ \textbf{Length} \rightarrow \textbf{Length}/N \end{cases}$$



5,6. LED array

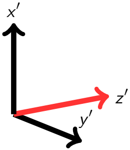
7. Vertical roller

8. Collection bucket

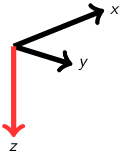
9. Camera stand

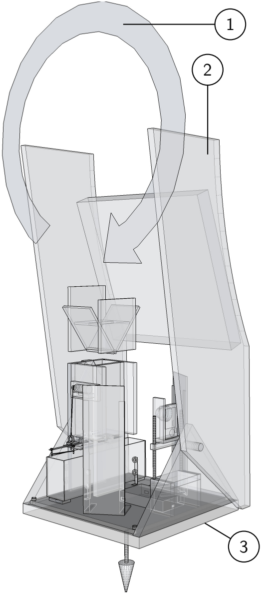
10. Data logger

Global co-ordinate system



Model co-ordinate system





Dimension	Length
Silo height	290mm
Internal width	150mm
Internal Thickness	100mm
Outlet width	30mm

- Two arrangements:
  - Flat-bottomed
  - Hopper with 30° half-angle
- Four centrifugal accelerations corresponding to 1g, 5g, 10g, 15g at the silo outlet.

Pressure pad calibration

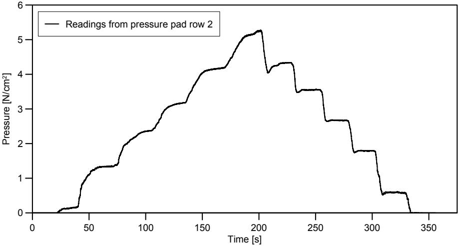


Figure 37: Typical data from a pressure pad calibration test

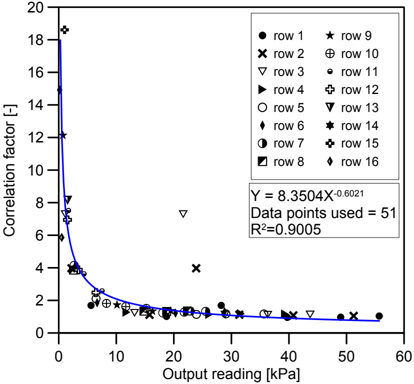


Figure 38: Calibration curve for blue pressure sensor using averaged data

Effect of hopper angle on gravity discharge rate Rose and Tanaka[1] reported the following correlation (pre-Beverloo[2]),

$$W = W_B F(\beta, \phi_d) \tag{4}$$

$$F(\beta, \phi_d) = (\tan\beta \tan\phi_d)^{-0.35} \qquad \text{for } \beta < 90 - \phi_d \tag{5}$$

$$F = 1 \qquad \text{for } \beta > 90 - \phi_d \tag{6}$$

where W_B is the discharge rate using the Beverloo correlation
ϕ_d can not yet be reliably predicted.

back

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[1] Rose and Tanaka, 1956.

[2] Beverloo, Leniger, and Velde, 1961.

Hour glass theory

$$W = C(K)\frac{\rho_b\sqrt{g^*}(l - kd)(D - kd)^{1.5}}{\sqrt{\sin\alpha}} \tag{7}$$

$$C(K) = \sqrt{\frac{1 + K}{2(K - 2)}} \tag{8}$$

$$K = \frac{1 + \sin\theta_i}{1 - \sin\theta_i} \tag{9}$$

Parameters

Parameter	Value
Wall normal stiffness [N/mm]	1e8
Wall shear stiffness [N/mm]	1e8
Wall friction coefficient [-]	0.4
Outlet width [mm]	20
Periodic thickness [mm]	5.95

Table 3: Wall parameters

Parameter	Value
Particle size [mm]	1.40 - 2.00
Material density [kg/m^3]	2655
Ball normal stiffness [N/mm]	1e7
Ball shear stiffness [N/mm]	1e7
Ball friction coefficient [-]	2.2

Table 4: Ball parameters

back

Density increase at increased gravities

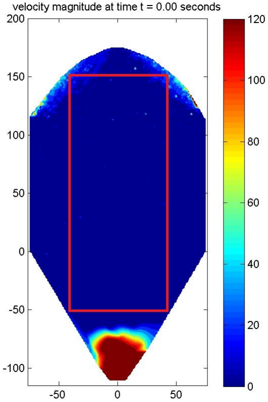

Figure 39: Region used to calculate bulk density in silo with 30 degree hopper

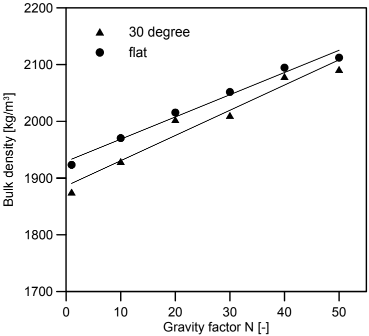

Figure 40: Bulk density at different gravities

back